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# Code No.: 1112 O

# VAŞAVI COLLEGE OF ENGINEERING (Autonomous), HYDERABAD B.E. I Year I-Semester (Old) Examinations, December- 2015

#### **Mathematics-I**

Max. Marks: 70 Time: 3 hours Note: Answer ALL questions in Part-A and any FIVE questions from Part-B

### Part-A (10 X 2=20 Marks)

1) Define linearly independent and linearly dependent vectors.

2) Find the eigen values of the matrix  $A = \begin{bmatrix} 3 & 1 \\ 4 & 0 \end{bmatrix}$ 

- 3) Write the Leibnitz test for testing the convergence of an alternating series.
- 4)  $\sum u_n + \sum v_n$  are two series of positive terms such that  $u_n \le v_n, \forall n$  compare the convergence or divergence of one series with reference to the nature of the other.
- 5) Define Center of curvature and write the formula for finding the center of curvature of a curve at a point (x,y) in Cartesian coordinates.
- 6) Discuss the symmetry property and asymptotic property of the curve

$$(y-2)[(x+3)^2+4a^2]=8a^3$$

- 7) Find  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial v}$ , when  $u = \log \left[ \frac{x^3 + y^3}{x + v} \right]$
- 8) Express Sin x Cos y as a polynomial in x,y of  $3^{rd}$  degree.
- 9) Evaluate the triple integral  $\int_{0}^{\pi} \int_{0}^{\pi/2} \int_{0}^{a} rSin\theta Sin\phi dr d\theta d\phi$

10) Find the area of the region bounded by the curve  $y = x^2$  and the line y = x

## Part-B (Marks: 5 X 10= 50) (All bits carry equal marks)

- 11) (a) Test the consistency of the system of equations 3x + 3y + 2z = 1; x + 2y = 4; 2x 3y z = 5and solve them if they are consistent
  - (b) Reduce the quadratic form  $3x^2 + 5y^2 + 3z^2 2yz + 2zx 2xy$  to the canonical form and specify the matrix of transformation.
- 12) (a) Define a series. Discuss the nature of convergence of the following series

i) 
$$\sum_{n=1}^{\infty} \sqrt{\frac{3^{n-1}}{2^{n-1}}}$$
 ii)  $\sum_{n=1}^{\infty} \frac{x^{n}}{1+x^{n}}$ 

(b) Define (i) alternating series (ii) Absolute convergence of a series (iii) Conditional convergence of a series. Also test the Absolute convergence of the series  $\sum_{0}^{\infty} \frac{(-1)^{n-1}}{n^2-1}$ 

- 13) (a) Find the radius of curvature at the origin of the curve  $y^2 = x^2 \frac{a+x}{a-x}$ 
  - (b) Show that the Evolute of the ellipse  $x = a\cos\theta$ ;  $y = b\sin\theta$  is  $(ax)^{\frac{2}{3}} + (by)^{\frac{2}{3}} = (a^2 b^2)^{\frac{2}{3}}$
- (a) Using Lagrange's method of undetermined multipliers, Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.
  - (b) If  $u = 2xy, v = x^2 y^2, x = r \cos \theta, y = r \sin \theta$ , find  $\frac{\partial(u, v)}{\partial(r, \theta)}$
- 15) (a) Evaluate  $\iint_{R} (x + y) dy dx$ , R is the region bounded by x = 0, x = 2, y = x, y = x + 2(b) Evaluate  $\int_{0}^{\infty} \int_{0}^{x} e^{-xy} y dy dx$  by changing the order-of the integration

(b) Endating  $J_0 = J_0$ (c) Endating  $J_0 = J_0$ (a) State Cayley-Hamilton theorem and verify with the matrix  $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$  and find  $A^{-1}$ 

(b) Examine the function  $f(x, y) = x^2 y^3$  where x + y = 35 for extreme values.

17) (a) Test the convergence of (i)  $1 + \frac{3}{7}x + \frac{3 \cdot 6}{7 \cdot 10}x^2 + \frac{3 \cdot 6 \cdot 9}{7 \cdot 10 \cdot 13}x^3 + \dots$ (ii)  $\sum \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{2 \cdot 4 \cdot 6 \dots (2n)} \frac{x^{2n+1}}{(2n+1)}$ 

(b) Trace the curve  $y^2 = \frac{x^2(1+x)}{2-x}$